SIGCSE Bulletin, Vol. 25, No. 1, March 1993

Integration of Discrete Mathematics Topics into the Secondary Mathematics Curriculum using Mathematica - A Summer Institute for High School Teachers

Dale A. Schoenefeld
Dept. of Mathematical and Computer Sciences
The University of Tulsa
Tulsa, Oklahoma 74104-3189
dschoen@euler.mcs.utulsa.edu

Roger L. Wainwright
Dept. of Mathematical and Computer Sciences
The University of Tulsa
Tulsa, Oklahoma 74104-3189
rogerw@penguin.mcs.utulsa.edu

ABSTRACT

The role of discrete mathematics in secondary mathematics courses has become an extremely important issue in recent years and has been addressed extensively by several national organizations. This paper discusses a summer Institute in discrete mathematics for high school teachers. A detailed outline is presented. Our survey indicated the participants of the Institute had "none" to "vague" prior exposure to 71% of the discrete mathematics topics presented in the course. The significant difference between this Institute and possibly others is the use of Mathematica as an integral part of the Institute. Mathematica provides an additional reinforcement of the material presented, especially in the area of graph theory and combinatorics. Mathematica can be a powerful motivating force and problem solving tool for high school students in their development of mathematical skills. We are encouraged by the success of this Institute in providing a needed service in our community. We hope the work reported in this paper will encourage others to develop similar Institutes in their communities.

1. Introduction

The National Research Council (NRC) [8,9], the Mathematical Association of America (MAA) [4], and the National Council of Teachers of Mathematics (NCTM) [6,7] have all recently produced reports representing efforts to improve the quality of school mathematics by providing a broad framework to guide reform. These documents are partially in response to studies like *A Nation at Risk* [5]. The NCTM Curriculum and Evaluation report [6] envisions what the school mathematics curriculum should include in terms of content priority and emphasis and provides an elaboration of the appropriate ways of evaluating student learning and school programs in mathematics. The MAA report [4] deals directly with the changes recommended in a collegiate mathematics curriculum designed for the preparation of teachers and identifies the mathematical knowledge, experiences and skills necessary to enable teachers to teach the program described by the NCTM Curriculum and Evaluation report. In the NCTM Curriculum and Evaluation report [6], the Standard 12: Discrete Mathematics section states that: "In grades 9-12, the mathematics curriculum should include topics from discrete mathematics so that all students can -

- represent problem situations using discrete structures such as finite graphs, matrices, sequences, and recurrence relations;
- represent and analyze finite graphs using matrices;
- develop and analyze algorithms;
- solve enumeration and finite probability problems;

and so that, in addition, college-intending students can

- represent and solve problems using linear programming and difference equations;
- investigate problem situations that arise in connection with computer validation and the application of algorithms."

The report concludes, "it is crucial that all students have experiences with the concepts and methods of discrete mathematics."

Almstrum [1] explores the role of mathematics in both undergraduate and pre-college computer science curricula, noting the importance of discrete mathematics among other topics. Taylor *et .al* [11] reports on the accreditation and certification standards for secondary computer science teachers, where, among other topics, discrete mathematics is considered very important. Cohen *et .al* [2] reports on how vital it is to the future of our nation to motivate high school students to excel in scientific studies. The authors believe equipping high school teachers to incorporate discrete mathematics topics into existing courses is one important way this can be done.

2. Motivation

No secondary mathematics course is taught in the Tulsa metropolitan region that is dedicated to discrete mathematics concepts. Furthermore, a survey of the education records of all 28 secondary mathematics teachers in a representative sample of two local area public schools and one private school reveals that none have taken a formal college course titled "discrete mathematics" or "discrete structures".

Some discrete mathematics concepts may be included in existing curriculum in our local area high schools without the existence of formal dedicated courses. However, our investigation concluded that this was a rare occurrence. The intensity of the endorsement and the promotion of discrete mathematics concepts by the Na-

ACM-24thCSE-2/93-IN,USA

tional Council of Teachers of Mathematics and the Mathematical Association of American coupled with the lack of formal background of local secondary mathematics teachers, prompted us to seek funding support in order to offer a discrete mathematics summer institute specifically for secondary mathematics teachers. The authors have had extensive experience teaching collegiate discrete mathematics and organizing other summer institutes for high school teachers [12,13].

3. Institute Objectives

The Institute's intent was to insure that the discrete mathematics standards that are being promoted by the NCTM and the MAA are reflected in the mathematical awareness and preparation of secondary mathematics teachers.

The primary objective of the Institute was to provide an opportunity for selected secondary (9-12) teachers to become topic literate and pedagogically proficient with the mathematics concepts contained in the Discrete Mathematics portion (Standard 12) of the NCTM Curriculum and Evaluation report [6]. The recommendations are generally echoed and replicated in the Discrete Processes portion (section IV, subsection 6) of the MAA report [4]. Although one desired outcome of the Institute was that secondary teachers be prepared, and be inclined, to introduce a separate secondary course in discrete mathematics, it was not the intent of the Institute to specifically encourage such a course. Rather, the orientation leaned toward identifying topics and alerting teachers to supporting curriculum materials that encourage the integration of discrete mathematics topics throughout the high school mathematics curriculum.

A second objective of the Institute was to review commercial curriculum modules and to design new curriculum modules that will facilitate the integration of discrete mathematics topics into traditional mathematics classes. Each module was oriented toward a single application. The module was focused on the problem statement, the body of applicable mathematics useful in formulating a problem solution, and a specific statement of solution to the problem. Several modules were obtained from the Consortium for Mathematics and its Applications (COMAP) funded by the National Science Foundation.

A third objective of the Institute was to demonstrate and emphasize the relationship between technology and mathematics. The objective was met in the context of the role played by computing software and hardware in support of the instruction and practice of discrete mathematics. In particular, this was accomplished by using Mathematica as an integral part of the course. Participants were given daily Mathematica assignments. Mathematica is a registered trademark of Wolfram Research, Inc. [14].

4. Institute Participants and Administrative Issues

Nineteen students participated in the Discrete Mathematics Institute. They represented twelve different school districts (including private schools) from the local metropolitan area and surrounding region. Several students commuted over forty miles one way daily. Thus one of our goals that this be a regional Institute was achieved. All participants were mathematics teachers except for one physics teacher. Each student had the opportunity to enroll in the course for academic credit (three graduate hours). Our University provided a tuition scholarship for each participant. In addition each participant was provided a $500 stipend upon the completion of the course whether they took the course for credit or not. The textbook and all other materials were provided at no cost to the participant. *Discrete Mathematics*, by Dossey *et .al* [3] was the textbook for the Institute. We choose this text because it seemed suitable for possible adoption as a high school textbook in discrete mathematics. As a supplemental textbook we used *Implementing Discrete Mathematics - Combinatorics and Graph Theory with Mathematica* by Skiena [10].

We asked participants to indicate their level of awareness (prior to the Institute) of 45 different discrete mathematics topics. The results are shown in Table I. These topics were all presented in the Institute in the order given in Table I. A frequency count from the 19 participants is shown for each topic. In addition the responses were weighted 0, 1, 2, and 3 respectively so that a weighted average for each topic could be determined. Notice that for 32 of the 45 topics the average was less than one. That is, for 71% of the material presented in the Institute, the participants, on the average, had "none" to "vague" prior knowledge of the topic.

5. Course Content Emphasizing Mathematica

The significant difference between this Institute and possibly others was the use of Mathematica as an integral part of the Institute. Nearly every topic covered in discrete mathematics has a supporting function implementation in Mathematica. Daily assignments in Mathematica were given. The authors feel very strongly that Mathematica provides an additional reinforcement of the material presented, especially in the area of graph theory and combinatorics. Furthermore, Mathematica can be a powerful motivating force and problem solving tool for high school students during their development of intuition and mathematical skills. In several years, it should be common place to see Mathematica or a similar tool in the high schools. When this happens our Institute participants will be prepared to immediately incorporate this into their courses.

The Institute ran for four weeks, Monday through Thursday. Each day consisted of lectures from 8:30 to 11:30 am with a break of 20 minutes. A two hour lab assignment was given each day consisting of Mathematica exercises as well as regular homework problems to solve by hand. The afternoon was basically an open laboratory session where the participants could come and go. Most stayed several hours each day completing the work. A computer laboratory consisting of twelve Apollo work stations was reserved exclusively for the participants of the Institute. With an open laboratory, twelve was more than enough work stations so that each participant could have his own machine. A full time assistant with a Mathematica background was on duty during the laboratory hours. Homework was assigned each day, but was not collected. We encouraged participants to work together on the problems, although each one made sure they understood the problems. One of the goals of the Institute was to bring together teachers from the community who taught mathematics and allow them to interact. The homework assignments provided an excellent forum for doing this. A room with a large conference table was reserved, in addition to the classroom and computer laboratory, specifically for the participants to work on problems and/or relax and interact with one another. This worked very well. Many of the participants learned from each other.

Table II gives a detailed summary of the topics covered each day, and the Mathematica commands that were appropriate for the day's lecture and associated laboratory assignment. Two examinations were given. The examinations were based on the course lectures and homework. In addition, a comprehensive Mathematica laboratory assignment was given during the last week. Each participant completed the Mathematica laboratory independently and turned in the assignment for evaluation.

6. Conclusions

In recent years there has been increased attention and use of discrete methods in mathematics and other disciplines. Problems from such diverse fields as business, industry, government, the natural and social sciences require finite processes and discrete techniques. The development of computer technologies has made possible exploration of practical problems which were otherwise too large or complex for analysis. Although the area called discrete mathematics is not yet fully defined, the areas of mathematics commonly grouped under this term are an essential part of the mathematical preparation of secondary teachers, and the prerequisite training of computer science majors.

In summary, Table I indicates the level of prior knowledge in the traditional topics of discrete mathematics among our participants was virtually non-existent. From Table II it is clear that the participants were exposed to a rigorous four week course. A survey taken on the last day of class indicates that the level of satisfaction among the participants was extremely high. Furthermore, the survey shows the participants were extremely pleased that Mathematica was integrated into the course, and indicated this was very significant in their learning experience. The survey also indicates the participants have a positive attitude and deeper understanding of the importance of discrete mathematics topics in secondary mathematics courses.

We are encouraged by the success of this Institute in providing a needed service in our community. All materials used in this Institute are available through the authors to any interested instructor. We hope the work reported in this paper will encourage others to develop similar Institutes in their communities.

Acknowledgements

This project was supported by the State Regents for Higher Education under the U.S Congress Dwight D. Eisenhower Mathematics and Science Act of 1988.

References

[1] Almstrum, V.L., "The Relationship between Pre-College Mathematics and the Undergraduate Computer Science Curricula", *SIGCSE Bulletin*, Vol. 23, No. 1., March, 1991, pp.124-129.

[2] Cohen, M., Foster, M., Kratzer, D., Malone, P. and Solem, A., "Get High School Students Hooked on Science with a Challenge", *SIGCSE Bulletin*, Vol. 1, No. 1, March 1992, pp.240-245.

[3] Dossey, Otto, Spence, Vanden Eynden, *Discrete Mathematics*, Scott, Foresman and Company, 1987.

[4] Mathematical Association of America. (1991) *A Call For Change, Recommendations for the Mathematical Preparation of Teachers of Mathematics*, Leitzel, James R. C., Editor.

[5] National Commission on Excellence in Education: *A Nation at Risk*

[6] National Council of Teachers of Mathematics. (March 1989) *Curriculum and Evaluation Standards for School Mathematics*.

[7] National Council of Teachers of Mathematics. (March 1991), *Professional Standards for Teaching Mathematics*.

[8] National Research Council, Board on Mathematical Sciences. (1990) *Renewing U.S. Mathematics, A Plan for the 1990s*.

[9] National Research Council, (January, 1989). *Everybody Counts: A Report to the Nation on the Future of Mathematics Education*.

[10] Skiena, S., *Implementing Discrete Mathematics - Combinatorics and Graph Theory with Mathematica*, Addison-Wesley, 1990.

[11] Taylor, H.G. and Martin, C.D., "The Impact of New Accreditation and Certification Standards for Secondary Computer Science Teachers on University Computer Science Departments", *SIGCSE Bulletin*, Vol. 1, No. 1, March 1992, pp.235-239.

[12] Wainwright, R. L. and Schoenefeld, D. A., "Advanced Placement Plus in Computer Science, A Summer Institute at The University of Tulsa," *SIGCSE Bulletin*, Vol 19, No. 1, Feb, 1987, pp. 84-91.

[13] Wainwright, R.L., "Introducing Functional Programming in Discrete Mathematics", *SIGCSE Bulletin*, Vol. 1, No. 1, March, 1992, pp 147-152.

[14] Wolfram, Steven , *Mathematica, A System for Doing Mathematics by Computer, Second Edition, Addison-Wesley*, 1991.

Table I
The Level of Participant Awareness (by Topic) Prior to the Start of the Institute. (19 Participants)

Topics	Prior Awareness (Frequency Count)				
	0 None	1 Vague	2 Moderate	3 Substantial	**Average**
1. Mathematica	8	7	4	0	**0.8**
2. Matrices	0	1	10	8	**2.4**
3. Set Operations	0	4	11	4	**2.0**
4. Equivalence Relations	1	6	8	4	**1.8**
5. Mathematical Induction	0	6	9	4	**1.9**
6. Logic and Proofs	1	2	12	4	**2.0**
7. Statements and Connectives	1	5	10	3	**1.8**
8. Logical Equivalence	1	7	10	1	**1.6**
9. Methods of Proof	0	5	10	4	**1.9**
10. Graphs and their Representation	3	7	7	2	**1.4**
11. Euler and Hamiltonian Circuits	12	4	3	0	**0.5**
12. Directed Graphs	11	7	1	0	**0.5**
13. Traveling Salesman Problem	10	9	0	0	**0.5**
14. Coloring a Graph.	8	10	1	0	**0.6**
15. Shortest Path and Distance	12	6	1	0	**0.4**
16. Network Flows and Cuts	14	4	1	0	**0.3**
17. Maximal Flow Algorithm	15	3	1	0	**0.3**
18. Hall's Theorem in Matching	17	2	0	0	**0.1**
19. Maximal Matching Algorithm	17	2	0	0	**0.1**
20. Trees	9	9	1	0	**0.6**
21. Algorithms for Minimum Spanning Trees	16	3	0	0	**0.2**
22. Traversals of Trees	13	5	0	1	**0.4**
23. Binary Search Trees	10	8	0	1	**0.6**
24. Iteration versus Recursion	13	3	2	1	**0.5**
25. Induction as a basis for Recursion	13	4	2	0	**0.4**
26. Coding Theory	13	5	1	0	**0.4**
27. Hamming Error Correcting Codes	16	1	2	0	**0.3**
28. Convolution Error Correcting Codes	17	1	1	0	**0.2**
29. Permutations	1	6	10	2	**1.7**
30. Combinations	1	6	10	2	**1.7**
31. Solving "MISSISSIPPI" Type Problems	5	5	6	3	**1.4**
32. Solving Problems using "Barricades"	12	6	1	0	**0.4**
33. Recurrence Relations	15	1	3	0	**0.4**
34. First-Order Linear Diff. Equations	12	5	2	0	**0.5**
35. Second-Order Homo. Linear Diff. Equ.	12	5	2	0	**0.5**
36. Insertion Sort	8	8	3	0	**0.7**
37. Selection Sort	9	6	4	0	**0.7**
38. Mergesort	7	9	3	0	**0.8**
39. Quicksort	12	7	0	0	**0.4**
40. Logic Gates	12	6	1	0	**0.4**
41. Creating Combinatorial Circuits	13	5	1	0	**0.4**
42. Finite State Machines	14	5	0	0	**0.3**
43. Linear Programming	4	8	5	2	**1.3**
44. Games Theory - (Min-Max)	13	5	1	0	**0.4**
45. Alpha Beta Pruning	17	2	0	0	**0.1**

Table II
Discrete Mathematics Institute Course Outline

Day	Lecture Topics	Mathematica Commands*
Mon	Motivation, Introduction to Mathematica, matrices, solving linear systems	Det[m], Inverse[m], LinearSolve[m,b], RowReduce[m], Solve[eqns,vars], TableForm[list]
Tue	Equivalence relations, mathematical induction, logic and proofs	
Wed	Graphs and their representations, Euler and Hamiltonian circuits, directed graphs	Graph[list,list], ShowLabeledGraph[g], ShakeGraph[g,d] Edges[g], Vertices[g], EulerianCycle[g], EulerianQ[g] HamiltonianCycle[g], HamiltonianQ[g],
Thu	Traveling salesperson problem, coloring a graph, shortest path and distance	TravelingSalesman[g], ChromaticNumber[g], ChromaticPolynomial[g,z], VertexColoring[g], ShortestPath[g,start,end], AllPairsShortestPath[g]
Mon	Network flows and cuts, maximal flow algorithm	NetworkFlow[g,source,sink], NetworkFlowEdges[g,source,sink]
Tue	Hall's matching theorem, maximal matching algorithm, matching as a flow	BipartiteMatching[g], BipartiteQ[g], TwoColoring[g], MaximalMatching[g], StableMarriage[mpref,fpref]
Wed	Trees, minimum spanning trees, traversal of trees	MaximumSpanningTree[g], MinimumSpanningTree[g], NumberOfSpanningTrees[g], ShortestPathSpanningTree[g,v]
Thu	Midterm examination	
Mon	Binary search trees, iteration versus recursion, induction versus recursion	BinarySearch[list,k,f]
Tue	Coding theory, error correcting Hamming codes, error correcting convolution codes	
Wed	Fundamental counting principle, permutations and combinations	Permutations[list], PermutationQ[list], Permute[list,p]
Thu	Recurrence relations, first and second order linear difference equations	RSolve[eqn,a[n],n]
Mon	Insertion sort, selection sort, mergesort, quicksort, order of algorithms	Sort[list], SelectionSort[list,f]
Tue	Logic gates, combinatorial circuits, finite state machines	
Wed	Linear programming, mathematics of finance, game theory	ConstrainedMax[f,{inequalities},{x,y,...}] ConstrainedMin[f,{inequalities},{x,y,...}] LinearProgramming[c,m,b]
Thu	Final examination	

*Most commands are in the DiscreteMath/Combinatorica.m package as documented in [10]. Others are either in the kernel and are documented in [14] or are in other packages of the standard DiscreteMath collection of Mathematica 2.0.